

Quartet correlations in $N=Z$ nuclei induced by realistic two-body interactions

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Two variational quartet models previously employed in a treatment of pairing forces are extended to the case of a general two-body interaction. One model approximates the nuclear states as a condensate of identical quartets with angular momentum $J = 0$ and isospin $T = 0$ while the other let these quartets to be all different from each other. With these models we investigate the role of alpha-like quartet correlations both in the ground state and in the lowest $J = 0$, $T = 0$ excited states of even-even $N = Z$ nuclei in the sd -shell. We show that the ground state correlations of these nuclei can be described to a good extent in terms of a condensate of alpha-like quartets. This turns out to be especially the case for the nucleus ^{32}S for which the overlap between this condensate and the shell model wave function is found close to one. In the same nucleus, a similar overlap is found also in the case of the first excited 0^+ state. No clear correspondence is observed instead between the second excited states of the quartet models and the shell model eigenstates in all the cases examined.

1 INTRODUCTION

As shown for the first time by A. Bohr et al. [1], many properties of open shell nuclei, such as the energy gap in the spectra of even-even nuclei and the moment of inertia of deformed nuclei, can be simply understood in the framework of the BCS approach [2]. Within this approach, the nucleons with energies close to the chemical potential are supposed to be correlated in Cooper pairs which act coherently in the form of a pair condensate. The term condensate stands here for a many-body wave function of the type $A\{\phi(x_1, x_2)\phi(x_3, x_4)\dots\}$ in which all Cooper pairs are represented by the same two-body wave function $\phi(x_i, x_{i'})$, with x_i denoting the positions and the spins of the nucleons, and A is the antisymmetrization operator. Such a condensate wave function, usually formulated in the configuration space, is commonly called a particle-number projected- BCS (PBCS) state, while the standard BCS state is a particular superposition of condensates with various pair numbers.

An interesting question, addressed for long time in nuclear structure, is whether in nuclei there are states which could be characterized in terms of alpha-type quartets acting coherently in the form of a condensate. In analogy with the PBCS case, this condensate would have the form $A\{\psi(y_1, y_2, y_3, y_4)\psi(y_5, y_6, y_7, y_8)\dots\}$, where all the alpha-like quartets are described by the same four-body wave function $\psi(y_1, y_2, y_3, y_4)$, with y_i denoting the positions, the spins and the isospins of the nucleons. By alpha-like quartet we mean a system of two neutrons and two protons coupled to the quantum numbers of an alpha particle, i.e. total isospin $T = 0$ and total angular momentum $J = 0$. It is worth stressing that the alpha-like quartets are not alpha particles.

The possibility of an alpha-like condensation in nuclei was mentioned more than 50 years ago in connection with proton-neutron pairing correlations [3–5]. To describe the ground state of the isovector pairing Hamiltonian, Flower and Vujici [4] proposed a BCS-like function built in terms of alpha-like quartets. The variational calculations with this trial state are complicated and, to our knowledge, they were never applied to realistic nuclear systems. A more general BCS-like ansatz, which contains both alpha-like quartets and pairs, was employed by Bremond and Valatin [5]. To make the calculations feasible, they considered only quartets built by two neutrons and two protons sitting on the same single-particle state, an assumption which leaves aside important correlations [4]. Similar alpha-like quartet models were used later on in refs. [6–8].

More general alpha-like quartet models, in which the neutrons and the protons which form the quartet are not constrained on the same orbital, have been proposed recently for realistic pairing Hamiltonians [10–16]. In refs. [14, 15] it was shown that the ground states of realistic isovector and isovector-isoscalar pairing Hamiltonians can be described with high accuracy as a condensate of collective quartets built by two neutrons and two neutrons coupled to $T = 0$. This approach, called quartet condensation model (QCM), conserves exactly the particle number and the isospin and, for degenerate states, it is equivalent to the exactly solvable $\text{SO}(5)$ and $\text{SU}(4)$ models for isovector and isovector-isoscalar pairing interactions [9]. Among the most important achievements of the QCM approach we mention

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that, (1), it has been proven that the isovector pairing acting on self-consistent Skyrme-HF mean fields can reproduce rather well the mass dependence of Wigner energy [12] and, (2), it has shown that the isovector and isoscalar pairing correlations coexist for any isovector and isoscalar pairing forces [14, 15]. This result is in agreement with the exactly solvable models but at variance with the predictions of standard HFB calculations [17, 18].

The question we want to address in this paper is whether the alpha-like condensation remains a reasonable ansatz for a realistic two-body force more general than pairing. The alpha-like condensation in the ground state of $N = Z$ nuclei was investigated in refs. [19, 20] by performing realistic calculations with two-body shell-model interactions in the fp shell. In these calculations the Pauli principle was exactly fulfilled only for systems up to two quartets while for the heavier systems a quasi-bosonic approximation was used. The purpose of this paper is to investigate the alpha-like correlations generated by realistic two-body interactions by employing variational quartet models which always preserve exactly the Pauli principle. We shall adopt two different models. One model approximates the nuclear states as a condensate of identical quartets with angular momentum $J = 0$ and isospin $T = 0$ while the other let these quartets to be all different from each other. The analysis will concern not only the ground state but also the lowest $J = 0, T = 0$ excited states. As an application we shall examine nuclei in the sd -shell. Nuclei in this shell have been the object of a recent analysis within a formalism of quartets [21]. This analysis was based on configuration interaction calculations carried out in a space of quartets of various J and T . As a basic difference with respect to the present work, in ref. [21] a “static” approach was adopted which consisted in extracting the quartets from the low-lying spectra of four nucleon systems outside the ^{16}O core and therefore keeping them fixed throughout all calculations. In the present work, instead, we shall adopt a “dynamical” approach and construct the quartets variationally for each nucleus. As we shall see in the following, this will allow a better understanding of the quartets correlations in the nuclear states.

2 FORMALISM

To investigate the quartet correlations we employ the most general two-body Hamiltonian commonly used in shell model calculations, i.e.,

$$H = \sum_i \epsilon_i N_i + \sum_{i,i',k,k',J,T} V^{JT}(ii';kk') [\mathcal{A}^{+JT}(i,i') \tilde{\mathcal{A}}^{JT}(k,k')]^{(0,0)}. \quad (1)$$

In the first term ϵ_i and N_i are, respectively, the energy and the particle number operator corresponding to the single-particle state $i = \{n_i, l_i, j_i\}$, where we have used the standard notation for the quantum numbers which label the spherical single-particle states. The Coulomb interaction between the protons is not taken into account, so the single-particle energies of protons and neutrons have the same values. The second term in eq. (1) is the two-body interaction written in terms of particle-particle operators

$$\mathcal{A}_{J_z, T_z}^{+JT}(i, i') = \frac{1}{\sqrt{(1 + \delta(i, i'))}} [a_i^+ a_{i'}^+]_{J_z, T_z}^{J, T}, \quad (2)$$

where J, T are the angular momentum and the isospin of the pair, respectively. The other pair operator in eq.(1) has the standard definition $\tilde{\mathcal{A}}_{J_z, T_z}^{JT}(i, i') = (-1)^{J-J_z+T-T_z} \mathcal{A}_{-J_z, -T_z}^{JT}(i, i')$. The notation $(0,0)$ in the second term of the Hamiltonian, as well as in the eq.(3) below, means that the two pair operators are coupled to total $J = 0$ and $T = 0$.

The Hamiltonian (1) will be employed here to investigate a certain class of 0^+ states, namely those which can be expressed in terms of collective alpha-like quartets. A collective alpha-like quartet is defined as

$$Q^+ = \sum_{i,i',k,k',J,T} x_{ii'kk';J,T} [\mathcal{A}^{+JT}(ii') \mathcal{A}^{JT}(k,k')]^{(0,0)}. \quad (3)$$

This collective quartet is more general than the ones that we have used previously in the case of pairing forces since it is built by all possible isovector ($T = 1$) and isoscalar ($T = 0$) non-collective pairs which can be formed. It should be observed that, by definition, the alpha-like quartet operator (3) is not a boson operator and does not represent an alpha particle.

With the collective quartet (3) we construct the quartet condensate model (QCM) state

$$|QCM\rangle = (Q^+)^{n_q} |-\rangle, \quad (4)$$

where n_q is the number of quartets. In the applications discussed in this study, the quartets will be built only with the valence nucleons which move outside a double magic core. This is represented by the vacuum state $|-\rangle$. The

main issue that we will address is to what extent the trial state (4) can represent the correlations generated by the two-body Hamiltonian (1) in the ground state of even-even $N = Z$ systems.

It is worth mentioning that, since the quartet (3) is not a boson, the state (4) is not a boson condensate. Here the term "condensation" has the same meaning as "pair condensation" in BCS-like theories: a product of many-body substructures (pairs in BCS, quartets in QCM) which are all in the same many-body state.

The state (4) depends on the mixing amplitudes x which define the collective quartet. These amplitudes are determined variationally by minimizing the expectation value $\langle QCM|H|QCM\rangle$ under the constraint $\langle QCM|QCM\rangle = 1$. To calculate the average of the Hamiltonian and the norm we apply standard many-body techniques.

In addition to the quartet condensate (4), we will also investigate a more sophisticated approximation which consists in representing the ground state of an even-even $N = Z$ nucleus as a product of collective distinct quartets

$$Q^{(d)+} = \sum_{i,i',k,k';J,T} x_{ii'kk';J,T}^{(d)+} [\mathcal{A}^{+JT}(i,i')\mathcal{A}^{+JT}(k,k')]^{J=0,T=0}. \quad (5)$$

The quartet model (QM) state that is constructed in this case is

$$|QM\rangle = Q^{(1)+}Q^{(2)+} \dots Q^{(n_q)+}|- \rangle. \quad (6)$$

A state of the form (6) was used recently to explore the quartet correlations associated with pairing forces both for like-particle [22] and proton-neutron systems [13, 16]. For the latter systems the collective quartets (5) contained only ($T = 1, J = 0$) and ($T = 0, J = 1$) pair operators.

The calculations with the QM state are more demanding than those within QCM because the number of parameters which have to be determined is n_q times larger than in the case of QCM. Owing to that, within QM, we do not construct all parameters at once through a direct minimization, as in QCM, but rather proceed through an iterative variational procedure which consists of a sequence of basic steps. At each step, we optimize the structure of a given quartet $Q^{(\rho)+}$ by searching for those coefficients x of this quartet which guarantee the minimum energy of the state (6). This is done by diagonalizing the Hamiltonian in a space formed by states of the type (6) where the quartet $Q^{(\rho)+}$ has been replaced by the uncorrelated quartets $[\mathcal{A}^{+JT}(i,i')\mathcal{A}^{+JT}(k,k')]^{J=0,T=0}$ while the other quartets are kept "frozen". The procedure starts with an initial ansatz for the coefficients x of the quartets and goes on by rotating the index ρ among all the n_q indices up to convergency of the energy. More details about this procedure can be found in ref. [22].

The present study will deal not only with the ground state of even-even $N = Z$ systems but also with excited states. There are many ways, in principle, in which excited states can be constructed within the QCM and QM schemes. For instance, by analogy with BCS-type models, in which the excitations are associated with broken pairs, in the quartet models excitations could be built by breaking quartets. The two protons and two neutrons of a broken quartet could be coupled in various way in order to get excited states. We shall compare two quite different approaches. Within the QCM we shall search for excited states which keep the form of a condensate, namely

$$|0_n^+; QCM\rangle = (Q_n^+)^{n_q}|- \rangle. \quad (7)$$

The collective quartet Q_n^+ associated with the excited state 0_n^+ will be determined by minimizing the functional $\langle 0_n^+; QCM|H|0_n^+; QCM\rangle$ under two types of constraints: a), the normalization of the state $|0_n^+; QCM\rangle$ and, b), the orthogonality of this state with the ground state as well as with all previously determined excited states. Within the QCM scheme, then, the excited states will be constructed in sequence. Within the QM approach, instead, assuming as collective quartets those defining the QM ground state, we shall construct all excited $J = 0, T = 0$ states at once by diagonalizing the Hamiltonian in a space formed by all possible states of the type (6) where, in rotation, one collective quartet has been "broken" and replaced by the uncorrelated quartet $[\mathcal{A}^{+JT}(i,i')\mathcal{A}^{+JT}(k,k')]^{J=0,T=0}$. As a result of this procedure, then, the excited QM state will be a linear superposition of states which are, each of them, of the type (6). More details about this procedure can be found in ref. [22] where it was successfully tested in the case of like-particle pairing.

3 RESULTS AND DISCUSSIONS

In this section we shall employ the QCM and QM schemes to explore the alpha-like quartet correlations in the even-even $N = Z$ nuclei of the sd shell. Following standard configuration mixing shell model (SM) calculations we shall assume the ^{16}O as a core and we shall adopt the USDB interaction [23].

We start by discussing to what extent the ground state correlations of these nuclei can be represented by the QCM and QM states. The results of the quartet models calculations for the ground states are presented in Tables

I-II. In Table I we show the correlation energies $E_{corr} = E_0 - E_{tot}$, where E_{tot} is the total ground state energy of the interacting system while E_0 is the energy of uncorrelated state in the absence of the two-body interaction. The correlation energies predicted by QCM and QM are compared to the exact shell model (SM) results, given in the second column. In brackets we indicate, in percentage, the differences between the predictions of the quartet models and the SM results. In the same table we also show, as a reference, the correlations energy of ^{20}Ne for which the QCM and QM states coincide with the SM state. In the last two columns we give the overlaps between the SM states and the QCM/QM states.

It can be observed that the predictions of QCM and QM for the ground state correlation energies are rather similar. The deviations from the SM results have a maximum for ^{24}Mg and they are seen to decrease significantly in the heavier nuclei. As expected, the results of the dynamical QM approach applied here, in which the quartets are determined variationally for each nucleus, are significantly better than those which were found within the QM approach of ref. [21] where, as $J = 0, T = 0$ quartets, we assumed those describing the ground state ^{20}Ne . For example, in the case of ^{28}Si we observed a deviation of about 6.6% from the SM ground state energy while, in the present QM calculation, this deviation is seen to drop to 1.84%

The quality of the QCM results of Table I indicates that a significant part of the the ground state correlations of the even-even $N = Z$ sd shell nuclei can be represented by a condensate of alpha-like quartets. This is especially the case for the nucleus ^{32}S , for which the QCM and SM states have an overlap close to one.

TABLE I: Ground state correlation energies, in MeV, predicted within the QCM and QM approaches in comparison with the shell model (SM) results. In brackets we show the differences, in percentage, between the SM results and the quartet models predictions. In the last two columns we report the overlaps between SM and QCM/QM states.

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$E_{corr}(QM)$	$\langle SM QCM \rangle$	$\langle SM QM \rangle$
^{20}Ne	24.77	24.77	24.77	1	1
^{24}Mg	55.70	53.04 (4.77%)	53.24 (4.41%)	0.85	0.87
^{28}Si	88.75	86.52 (2.52%)	87.12 (1.84%)	0.86	0.90
^{32}S	122.51	122.02 (0.40%)	122.29 (0.18%)	0.98	0.99

To facilitate a better understanding of quartet correlations in these nuclei, in Table II we report the total energies, $E_{tot} = \langle QCM|H|QCM \rangle$, the energies associated with the two-body interaction, $E_{int} = \langle QCM|V|QCM \rangle$, the energy $E_Q = \langle -|QHQQ^+|- \rangle$ of the QCM quartet of each nucleus and, in the last three columns, the occupancies of the single-particle states referred to this quartet.

TABLE II: QCM results for total energies $E_{tot} = \langle QCM|H|QCM \rangle$ and the interaction energies $E_{int} = \langle QCM|V|QCM \rangle$. In the second column we give the total energies provided by SM calculations. In the 5th column we show the total energies $E_Q = \langle -|QHQQ^+|- \rangle$ and, in parenthesis, the corresponding interaction energies of the QCM quartet for each nucleus. In the last three columns we give the occupancies of the single-particle states referred to the same QCM quartet. All energies are in MeV.

	$E_{tot}(SM)$	$E_{tot}(QCM)$	$E_{int}(QCM)$	E_Q	$n_{d_{5/2}}$	$n_{s_{1/2}}$	$n_{d_{3/2}}$
^{20}Ne	-40.47	-40.47	-28.74	-40.47 (-28.74)	2.49	0.97	0.54
^{24}Mg	-87.10	-84.45	-60.49	-39.96 (-27.53)	2.99	0.53	0.48
^{28}Si	-135.84	-133.63	-93.35	-37.58 (-23.46)	3.53	0.24	0.23
^{32}S	-182.44	-181.96	-133.81	-37.54 (-23.33)	3.46	0.34	0.21

From Table II one can observe that in the multi-quartet systems the alpha-like quartets become less bound than in ^{20}Ne , as a result of Pauli blocking. One can also notice a smooth evolution of the structure of the QCM quartets when passing from ^{20}Ne to ^{28}Si . This manifests itself in a smooth increase of $n_{d_{5/2}}$ and a parallel decrease of the other occupation numbers. At ^{32}S , namely beyond the middle of the sd shell, one sees a break of this trend likely to be related to an increasing role of the Pauli principle.

To the extent that the alpha-like quartets can be considered as elementary degrees of freedom, one can represent in first approximation the energy associated with a system of n_q identical quartets as the sum of two terms. The first term is proportional to n_q and it accounts for the total energy of the system in the absence of any interaction among

the quartets. The second term is proportional to $n_q(n_q - 1)$ and it arises instead from a two-body interaction among the quartets. Under these assumptions and by adopting the quartet associated with ^{20}Ne as the reference quartet, the energy of the system can be therefore represented as

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q - 1)}{2} \times V(n_q), \quad (8)$$

with $E(1)$ being the energy of the one quartet system while $V(n_q)$ denotes the interaction energy between two quartets. By inserting in eq. (8) the energies E_{tot} provided by the QCM, one gets the values $V(2) = -3.51$, $V(3) = -4.07$ and $V(4) = -3.34$ (in MeV). These interaction energies appear to be small compared to the energy of the quartet and weakly depending on the particle number, properties which are emphasizing the "condensed" structure of QCM state (4). Particularly interesting is the fact that the interaction between two quartets turns out to be always attractive. This finding is in agreement with that of ref. [19] obtained in a similar analysis of realistic nuclei in the pf shell.

We do want to emphasize at this stage that the attractive/repulsive nature of the interaction among the quartets of a condensate is strongly depending on the nuclear interaction in use. To clarify this point we refer to the results which are reported in Table III of ref. [15]. There one finds, among the rest, the ground state correlation energies that are calculated in the QCM approach with a Hamiltonian which contains only the isoscalar and isovector pairing components of the USDB interaction that we employ in the present work. The corresponding total ground state energies are (in MeV) $E(^{20}\text{Ne}) = -31.69$, $E(^{24}\text{Mg}) = -60.00$ and $E(^{28}\text{Si}) = -82.40$. When inserting these values in eq. (8), with $E(1) = E(^{20}\text{Ne})$, one finds $V(2) = +3.38$ and $V(3) = +4.22$ (in MeV). The interaction among the quartets that comes out in this case is therefore repulsive.

It is worth noticing that a repulsive interaction between quartets can also be deduced in the case of the exactly solvable isovector-isoscalar $\text{SU}(4)$ pairing Hamiltonian [9]. Indeed, it can be easily seen that the exact ground state energy of an even number of proton-neutron pairs n_p in a degenerate level can be expressed by $E(n_q) = n_q E(n_q = 1) + g n_q(n_q - 1)/16$, where $n_q = 2n_p$ is the number of quartets and $-g(g > 0)$ is the strength of the isovector and isoscalar pairing forces. This expression, which is similar to eq.(8), shows that in this case there is a repulsive interaction between the quartets with strength $V = g/8$.

In general, the interaction among the quartets essentially results from two competing effects: on one side, the Pauli principle generates a repulsion among the quartets which becomes the more evident the more we fill the model space and, on the other side, the attraction among the nucleons caused by the nuclear force produces the opposite effect. Clearly, when passing from the simple isovector plus isoscalar pairing interaction to the full USDB Hamiltonian, the $J > 1$ components of the nuclear force have been such to turn the overall interaction among the quartets from repulsive to attractive by overwhelming the repulsion generated by the Pauli principle which prevailed instead in the pairing case.

We conclude this section by showing the QCM/QM excited 0^+ states which are calculated in the systems examined so far and comparing them with the SM results. In Table III we report the energies of the first two excited 0^+ states, $E_{0_1^+}$ and $E_{0_2^+}$, as calculated within the QCM, QM and SM approximations. For the first excited 0^+ state we also show the overlaps between the SM state and the QCM/QM states (it should be observed that by 0_1^+ we denote the first excited state and not the ground state). The excited states of ^{20}Ne shown in Table III are, by construction, the same in the QCM, QM and SM calculations.

TABLE III: The energies, in MeV, of the first and the second excited 0^+ states, $E_{0_1^+}$ and $E_{0_2^+}$, provided by QCM, QM and SM approaches. In the bracket are given the excitation energies relative to the ground state. In the last two columns are shown, for the first excited 0^+ state, the overlaps between the SM state and the QCM/QM states.

	$E_{0_1^+}(SM)$	$E_{0_1^+}(QCM)$	$E_{0_1^+}(QM)$	$E_{0_2^+}(SM)$	$E_{0_2^+}(QCM)$	$E_{0_2^+}(QM)$	$\langle SM QCM \rangle$	$\langle SM QM \rangle$
^{20}Ne	-33.77 (6.7)	-33.77 (6.7)	-33.77 (6.7)	-28.56 (11.91)	-28.56 (11.91)	-28.56 (11.91)	1	1
^{24}Mg	-79.76 (7.34)	-76.97 (7.47)	-78.00 (6.64)	-77.43 (9.67)	-70.85 (13.59)	-73.28 (11.36)	0.70	0.78
^{28}Si	-131.00 (4.84)	-126.91 (6.71)	-128.94 (5.27)	-128.51 (7.33)	-120.64 (12.99)	-125.01 (9.20)	0.65	0.78
^{32}S	-178.98 (3.46)	-178.04 (3.92)	-178.71 (3.51)	-175.04 (7.4)	-170.84 (11.12)	-173.71 (8.51)	0.95	0.99

The most remarkable results in Table III are the ones for ^{32}S . For this nucleus it can be seen that the QCM function which describes the first excited 0^+ state has an overlap close to one with the SM state. This means that in ^{32}S both

the ground state and the first excited 0^+ state can be represented to a high degree of precision as a condensate of four identical alpha-like quartets. A significant overlap with the SM state is also observed for the first excited 0^+ states of ^{24}Mg and ^{28}Si both for QCM and QM. Taking also into account that the energies predicted by the quartet models are not very different from the SM values, it appears that also in these nuclei the quartet degrees of freedom play an important role in the structure of the first excited 0^+ states.

The energies of the second 0^+ states predicted by the quartet models are given in the 6th and 7th columns of Table III. These states have not a significant overlap with the second excited 0^+ shell-model state or with other shell-model states at higher energy. In fact, they have a non-negligible overlap with many 0^+ shell-model states. As in other variational models, such as BCS or generating coordinate model, the QCM and QM states are specific variational ansatz constructed to describe a certain class of physical states and, as such, they are not expected to correspond necessarily to some exact eigenstate of the effective Hamiltonian. The QCM ansatz for the excited states bears a resemblance to alpha condensate states employed to describe certain 0^+ states close to the alpha particle emission threshold [24]. Whether QCM can describe or not such cluster states is an issue which requires further investigations and calculations in much larger model spaces than the ones employed here.

4 SUMMARY AND CONCLUSIONS

In this study we have extended the QM and QCM variational approaches, previously employed for a treatment of proton-neutron pairing forces, to the most general two-body interaction of shell-model type. Using these variational models we have shown that the ground states of even-even *sd*-shell nuclei with $N = Z$ can be described to a good extent as condensates of alpha-like quartets.

In the framework of the same quartet models we have also analyzed the excited 0^+ states. We have found that the first excited 0^+ states predicted by the quartet models have a significant overlap with the first excited 0^+ states provided by shell model calculations. This is especially true for the nucleus ^{32}S for which this overlap (as well as that relative to the ground state) is close to one.

As far as the second excited 0^+ states are concerned, the states predicted by the quartet models have no significant overlaps with any shell model state. These states appear at high energies, some of them in the energy region of alpha particle emission threshold. However, it is not yet clear whether these states can be associated with the physical alpha cluster states predicted by alpha cluster models in this energy region. To clarify this issue one would need to perform calculations in much larger shell model spaces, what is beyond the scope of this study.

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